

Tōkyō, 11 Février 2008

LOGIC  
VS.  
INTELLIGENCE

**JEAN-YVES GIRARD**

## 1 — INTRODUCTION

- Can one *mechanise* intelligence ?  
**Leibniz** : *ratiocinator*.  
**Hilbert** : *reductionist* foundations.  
**AI** : a prothesis for the *true believers*.
- Success w.r.t. *rational*, predictable, activity :  
**Artificial instinct** : vision, motion, etc.
- Absolute failure w.r.t. *irrational*, i.e., creative activity :  
**Gödel' paradox** : *incompleteness*.  
**Turing** : undecidability of the *halting* problem.  
**Search** does not work, only *verification*.

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# I — THE CRISIS OF FOUNDATIONS

## 2 — THE XIX<sup>th</sup> CENTURY

- Change of **size** w.r.t. XVIII<sup>th</sup> century maths :  
**Functions**  $\log, \sin, \exp$  replaced with « **all functions.** »  
**3-dimensional space** replaced with **varieties**.
- Divergence w.r.t. intuition, emergence of **paradoxes** :  
**Geometry** : non Euclidian spaces.  
**Curve** without tangent (Weierstraß).  
**Curve** filling a surface (Peano).
- Doubts as to the **soundness** of mathematics :  
**Topology** : is a surface the same as a volume ?  
**Consistency** : are subdomains of maths compatible ?

### 3 — THE XX<sup>th</sup> CENTURY

- The myth of *triumphant science* (Jules Verne).  
**1914-18** : yet another triumph of science.  
**isms** : various « **scientific** » approaches to politics.
- One can *solve* all questions (!?)
- Computers prompt a come back of *scientism* :  
**H. Simon** : Kepler law « **rediscovered** » by computer.  
**Logics** of transparency.
- *Fails* at both levels :  
**Philosophical** : what is a good question ?  
**Technical** : doomed by *Gödel's paradox*.

## 4 — SET THEORY

- Cantor's work on *trigonometric series* :  
**Exception sets** : the origin of *set theory*.
- Turns into a *potential* unification of maths.
- *Creation* of the world in six days (Dedekind).

**Cardinals** : pure numbers  $\mathbb{N}$ .

**Algebra** : relatives and rationals  $\mathbb{Z}, \mathbb{Q}$ .

**Analysis** : real and complex numbers  $\mathbb{R}, \mathbb{C}$ .

**Geometry** : vector spaces, varieties  $\mathbb{R}^3$ .

**Banach spaces** :  $l^1, l^2, l^\infty$ .

**Operator algebras** :  $C^*$ -algebras, von Neumann algebras.

## 5 — THE SEVENTH DAY

- Paradoxes of set theory :

**Cantor ~ 1880** :  $\mathbb{R}$  not denumerable, yields :

**Russell (1902)** : define  $A := \{x; x \notin x\}$

$$A \in A \Leftrightarrow A \notin A.$$

- Russell's paradox is an *antinomy* (mortal).
- Need (*or pretext*) for ultimate foundations.
- *Hilbert's program* ~ 1925 : foundations of mathematics  
**Rigorous** thence  
**Mathematical** : the snake bites its own tail !

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## **II — HILBERT'S PROGRAM**



## 6 — A JIVARIST MATRIOSHKA

- *Intelligence* reduced to science ; *exit* poetry.
- *Science* reduced to mathematics ; *exit* physics.
- *Mathematics* reduced to language ; *exit* geometry.
- *Bureaucracy* as the paragon of intelligence.  
**Compulsive maniac** : checks fly wings.  
**Formal logician** : checks brackets.
- So extreme that it contains *its own demise* :  
**Philosophy** : « **proving** » a viewpoint, a controversial idea.  
**Refutation** of such a(n extreme) viewpoint thus possible.  
**Irrefragability** of Gödel's refutation.

## 7 — TRANSPARENCY

- Reduction of truth to *provability* or *consistency* :

*A true*  $\Leftrightarrow$  *A provable.*

*A true*  $\Leftrightarrow$  *A consistent.*

- Thence Hilbert's *consistency* program.
- Should yield transparency, the *underside* of reality ...  
**Easy** : look at the *list* of consistent questions !  
**Legibility** of the world, total and *immediate*.  
**Password ideology** : *decode* the appearances.
- *Commonsense* : the world is not transparent.
- A necessary thing, neither a *perversion of Man*,  
Nor an *ordeal of God*.

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## **III — GOEDEL'S PARADOX**

## 8 — THE 11<sup>th</sup> HOUR WORKER

- In order to refute the program, one must partly *fulfill* it !
- Continuation of Dedekind : *encoding* the language.  
**Chinese remainder theorem** codes *sequences*.  
**Gödel numbers** : enumeration of linguistic artifacts.
- Russell's antinomy becomes « **I am not provable.** »
- 1<sup>st</sup> *incompleteness* theorem (1931) :  
**Gödel formula** true, but *not provable*.
- More bureaucracy yields 2<sup>nd</sup> *incompleteness* thm. (1931) :  
**Consistent** : system cannot prove its own consistency.
- An army which proves *its own innocence* is ...

## 9 — TRUE BELIEVIERS STRIKE BACK !

- Paradoxes destroy belief, thence should be *disqualified*.
- Make it a sort of *nonsense* ( Gödel-Escher-Bach ) :  
**Gödel numbers** as a new Kabbalah.  
**Scientism** comes out of obfuscation.
- *Fix* it by making logic transparent :  
**Para-consistent logics** : *pas vu pas pris*.  
**Rubber checks** as paraconsistent economy.
- *Refute* it (the work of half-wits) :  
**Inconsistency** follows from *refutation* ; would thus  
**Reinforce** the theorem, hitherto the only *irrefragable* one !

## 10 — TURING'S COUP DE GRÂCE

- ~ 1936, Turing sets the foundations of *computers*.
- Incompleteness as *undecidability* of *halting* :  
**Bus of Roma** : does not arrive.  
**No way** to know it.
- *Not to know*  $\neq$  *To know that not*.
- Refutes the myth of *transparency* :
- Independently of *time/space* limitations :  
**Questions** need not have answers ...  
**Not even** incorrect ones.
- *A close shave !*

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# **IV — MECHANISING INTELLIGENCE ?**

## 11 — SUCCESSES AND LIMITS OF MECHANISATION

- By-product of encodings : *image processing*, etc.
- Efficient *verification*, e.g., word processing.
- Theorem « **provers** » can :
  - Assist** the mathematician.
  - Verify** , i.e., *formalise* proofs.
- Irrecoverable *failure* of :
  - Mechanistic** foundations.
  - Automatic provers** : completely inefficient.
- For « **feasible** » questions, is *finding* the same as *checking* ?

$$P \stackrel{?}{=} NP$$



## 12 — LOGIC AS FORMAL DUMBNESS

- Gödel, Turing : intelligence *cannot* be mechanised.
- In contrast with *repetition*, i.e., with :  
**Logic activity** : rational and dumb, checking brackets.
- What cannot be mechanised is *creativity* :  
**Bureaucrats** good at checking, limited thinkers.
- Creativity is a by-product of *irrationality* :  
**Prejudices** : jealousy, anger, the seven sins and worse !  
**Mistakes** consubstantial to intelligence.  
**Natural equilibrium** : intelligent beings have *limited power* !

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