KEIO, 10 Novembre 2015

TRANSCENDENTAL SYNTAX

JEAN-YVES GIRARD

The reconciliation of the *mathematical* and *philosophical* sides of logic thanks to *informatics*.

1 — LOGIC, A MEDIATION RATIONAL/IRRATIONAL

- ***Rational*** **from** ***ratio*** **(of a division): reduction to numbers.**

  Ratiocinator: (Leibniz) verge of irrationality ***(Kabbalah)*****.**
  Numerisation: sounds, images,. . . and omniscient robot.

- ***Semantics*** **as the XX$^{\text{th}}$ century Kabbalah;** ***scientism*****.**

  Realism: no distinction question/answer: same ***denotation*****.**
  Selfy: the word ***pipe*** **refers to a PIPE, i.e., another word.**
  God created the monkey in His own image.

- ***Transparentism*****: approach without mediation,** ***immediate*****.**

  Claim: one can ***always*** **answer, compare and predict.**
  Refuted: by ***incompleteness*** **(Richard/Gödel/Turing).**

- **Logic like** ***police*** **with bad cops: no** ***divide*** **rational/irrational.**

  Deductive aspects: ***implication*****, cheques on the unknown.**
  Pact: between reason and the demons of « déraison ».

René Magritte (1898–1967).

2 — TRANSCENDENTALISM

- **Semantic selfy *subjectivistic* since refuses *Subject*.**

  Subject part of logic because of *logos*.
  Search for presuppositions, *conditions of possibility*.
  Analytic philosophy no presupposition; hides *prejudice*.

- ***Three lights* and *four cardinal points* against prejudice.**

  1–Answers? *Analytic:* beyond discussion, but *meaningless*.
  2–Questions? *Synthetic:* controversial, convey *meaning*.
  3–Certainty? *Reasonable;* doubts remain *legitimate*.

- **Raw/Formatted: analytic/synthetic, untyped/typed.**
  Explicit/Implicit: *a posteriori/a priori*, cut-free/deductive.

	RAW	FORMATTED
EXPLICIT	$1-Constat$	$3-Usine$
IMPLICIT	$2-Performance$	$4-Usage$

I — WHAT IS AN ANSWER?

Keywords: analytic, untyped, computational.

3 — THE ANALYTIC, A.K.A. RAW

- ***Computers*** **yield answers, hard to ignore, even when *wrong*.**

  Sever relation to question, i.e., forget *meaning*.
  Analyticity: answer *autonomous*, beyond discussion.

- ***Kant:*** **analytic=logic, predicate part of the subject. *Outdated*.**

  Modern logic contains mathematics: not *analytic*.
  Green cats are green; but *popular* democracy not *popular*.

- **Dusted reading: everything *on the board*.**

  Excludes any sort of external reference, in particular:
  Infinity: the « etc. » not on the board.

- ***Pseudo-analyticity:*** **wrong claim to analyticity.**

  Semantics: infinite, external, a *reasoning* about analyticity.
  Photography: problem with *offscreen*. Belongs in *usine*.
  Verbatim: really analytic, used by cowards; *meaningless*.

4 — IMPLICIT VS. EXPLICIT

- ***Implicit:*** **what we don't have.**

  Dreams, lost horizons, infinity — what we can't *finish*.
  Or want to hold off: the origin of *abstraction*.

- ***Constat* vs. *performance*.**

  Table of logarithms: answers hanging like smoked herrings.
  Calculator: *indirect* answer, but much more efficient.

- **Opposition between two uses of ↲ *(return)* key.**

  Typewriter opens new line; *incremental*.
  Computer launches program; *destructive*.

- ***Give* him some tuna or *teach* him how to fish?**

  Explicit answer works for a single time.
  Implicit an. general: involves *explicitation*, performance.
  Better, if pupil skilled enough; may *diverge* otherwise.

5 — QUEST OF THE EXPLICIT

- **Logicist Pavlov's dog: explicit as *semantics* of implicit.**
  Fregean opposition *denotation/sense*, contents/form, etc.
  Essentialises distinction *data/programs*.
  Same nature on the board, i.e., on *computer*.
- **Explicit as *suspended* implicit: no need to proceed *further*.**
  Chessboard: $N = 2^{64} - 1 = 18446744073709551615$.
  Cheque: *cash* (implicit) or *display* on wall (explicit)?
- ***Everything* on the board: $27 + 37 = 64$ involves $27, 37$ and:**
  Program $+$; if mistaken for $\times$, $27 + 37 = 999$, still analytic.
- ***Check* that computation done according to book (program).**
  Pavlov's dog: *meta-analyticity*. Non analytic, since *external*.
  Performance performs itself, by *matching* opposite colours.
  Explicit: uncolored (black) links, not matchable.

6 — THE GREAT ANALYTIC DIVIDE

- The *pravdameter*, i.e., the machine to tell the truth.

  Kabbalah, Casanova, Leibniz: unfaithful codings.
  Computers: faithful (binary) codings.
  Rational numerology: the pravdameter as the Graal of AI.

- ***Procrastination:*** the pravdameter as a totalitarian fantasy.

  Babel Library (Borges): ***all*** books of a given format.
  Write, characterwise, ***anti-book*** $\neq$ from those consulted.

- Infinite book format ***paradoxical:*** Cantor vs. Turing.

  Constative books: Library impossible, can't even ***file*** books.
  `Kindle`: Library exists, but some characters cannot ***display***.

- ***Undecidability:*** states the impossibility of ***universal*** answer.

  No relation to questions; answers could be « wrong ».
  Cantor $\neq$ Turing: performance ***irreducible*** to constat.

7 — ONE CAN ANSWER EVERYTHING

- **The first subliminal slogan of *transparentism*.**

  X-rays of knowledge: the true reality beyond apparences.
  Realism, scientism: no doubts, down with Socrates!
  Totalitarism of NSA conspicuous in its claim to *neutrality*.

- **Undecidability: *reverse side of reality* pure fantasy, hence**

  Paradoxical: only known to gnostic sect. *Hermetism*.

- ***Hidden messages* in Nostradamus, Mallarmé, etc. baloney!**

  Hitchcock: MacGuffin, irrelevant secret message.
  Val Lewton: RKO producer, *suggestion* stronger than *vision*.

- **The *topsy-turvied* reverse « the first will be last ».**

  Escher: icon of « reverse side ». Gödel-Escher-Bach.
  Nonsensical view of logic: the man *always* telling lies.
  Superficial and transparentist: involves a *pravdameter*.

8 — PURE λ-CALCULUS

- **Best approximation to analyticity: *untyped*, no commitment.**

  Syntax: *terms* $x, \lambda xt, (t)u$**.**
  Constative: *normal* terms.
  Performance *rewriting* $(\lambda xt)u \rightsquigarrow t[u/x]$**.**

- ***Knitting* expressed by structural properties.**

  Church-Rosser redexes in two colours.
  Three performances equivalent.

- ***Forgetful* functor from typed (synthetic) systems (e.g.,** $\mathbb{F}$**).**

  Church-Rosser: compositionality of $\Rightarrow$ ***(associativity)*.**

- **Limitations:**

  Externality of performance: rewriting *redex* $\rightsquigarrow$ ***contractum*.**
  Functional commitment; unfit for parallelism.
  Non linearity: unfit for non-determinism.

9 — UNIFICATION AND MATCHING

- **Originates in Herbrand 1930; sort of analytic *η-expansion*.**

  Identity $A \between B \vdash A \between B$ **same as identities** $A \vdash A, B \vdash B$.
  Wire splits spontaneously into *subwires*.

- **Functional *terms:* wires. *Variables:* potential subwires.**

  Subwires activated by *matchings* $t = u$.
  Example: $a * y = x * b$**: common subwire** $a * b$.
  Example: $a * y = b * x$ **don't match, matching *fails*.**

- ***Unification:* search for *most general unifier* θ_0.**

  Unifier of t, u**:** substitution θ **s.t.** $t\theta = u\theta$.
  M.g.u. θ_0**:** any unifier θ uniquely writes as $\theta = \theta_0\theta'$.
  Unifiers for $a * y = a * x$, the $\theta_t(x) = \theta_t(y) = t$, m.g.u. θ_z.
  Matching: dynamic intersection $t \cap u$. ***Distinct*** **variables.**
  Failure: no unifier; not matchable = ***disjoint*.**

10 — STARS AND CONSTELLATIONS

- ***Star:*** $n \neq 0$ **terms** ***(rays)*** **with exactly the same variables.**

  Disjoint: rays pairwise not ***matchable***.
  Substitution: $[\![t_1, \ldots, t_n]\!]\theta := [\![t_1\theta, \ldots, t_n\theta]\!]$ **still a star.**

- ***Constellation:*** **finite set of stars.**

  Bound variables, i.e., local to each star.
  Rays of the (stars of the) constellation pairwise disjoint.

- ***Colours:*** **just a convenience, unary function letters.**

  Disjoint: come by ***complementary*** pairs.
  Pairs: `green`/`magenta`, `red`/`cyan`, `blue`/`yellow`.

- **Colours responsible for divide *constat/performance*.**

  Constative constellation: in black (no colour).
  Performance: elimination of colour, normalisation.
  GoI: analytic substrate of synthetic ***cut-elimination***.

11 — STRONG NORMALISATION

- ***Diagrams*** **of constellation:** ***tree*** **(connected/acyclic graph).**

  Vertices: stars (with repetitions). Infinitely many diagrams.
  Edges: formal equalities $t = u, t = u, t = u$.

- ***Actualisation*** **of a diagram:**

  Match underlying terms: $t = u$ becomes $t\theta = u\theta$.
  Failure of most actualisations; diagram *correct* otherwise.

- ***Strong normalisation:*** **knitting constat/performance.**

  1–Finiteness: all diagrams of size N, hence $\geq N$ fail.
  Excludes $[\![x, x]\!]$. *Undecidability:* no way to predict N.
  2–Openness: no *closed* correct diagram (with no *free* ray).

- ***Residual*** **star of correct diagram: its actualised *free* rays.**

  Normal form: constellation of *uncoloured* residual stars.
  Church-Rosser: two pairs of complementary colours.

12 — NON-DETERMINISM

- ***Non-determinism*** **in constellations allows matching rays.**

  Resolution: stars $\Gamma \vdash A$ **or** $\Gamma \vdash A$**: a fine mess.**
  Control: tries to fix bad analyticity, e.g., ***multiple matchings*****.**
  PROLOG: ***analytic*** **mingled with** ***synthetic*****, logic: fails.**
  Declarative programming: similar to ***analytic philosophy*****.**

- **Same problem with** π***-calculi*****.**

  Hesitate: parallel λ**-calculus or cheap linear logic?**

- **Matching rays can only represent** ***Alzheimer*****, NL-style.**

  Coordination: necessary in NP-style ***(satisfiability).***

- ***Non-deterministic*** **constellation:**

  Liberalised: matching rays allowed.
  Coherence: $\mathcal{S} \ddagger \mathcal{T}$**:** ***forbidden*** **substitutions.**
  Strong normalisation: ***self-incoherent*** **diagrams fail.**

13 — PARALLEL UNIVERSES

- ***Church-Rosser:*** **takes account of all *parallel* computations.**
- **Knitting with *usine:* one should take care of *additives*.**

  $A \,\&\, B$**: choose between « parallel universes »** A/B**.**
  Freshness: how do I know that my choice is not *biased*?
  If already in universe A**, I cannot see alternative** B**.**
  S-F analogue: movies style *The matrix*.
- **Herbrand:** ***formal*** **function** $f(t)$**, a variable unknown to** t**.**

  Herbrand boolean $\eta_{\mathcal{S}}$ **indexed by a substar of some** $\mathcal{T}$**.**
  Normalisation induces dynamic modification of booleans.
  Evolution of $\mathcal{T}$ **into** $\mathcal{T}'$ **induces parallel evolution** $\eta_{\mathcal{S}} \rightsquigarrow \eta_{\mathcal{S}'}$**.**
- $\eta_{\mathcal{A}\&\mathcal{B}}$**: boolean living « outside »** A/B**. Chooses** A**.**

  Cancellation with $\neg\eta_{\mathcal{A}\&\mathcal{B}}$**: only if behave in *same* way.**
  Arrival in $A \,\&\, B$**: not influenced by dichotimy** A/B**.**

II — WHAT IS A QUESTION?

Keywords: synthetic, typed, logical.

The Ouija board (∼ 1890).

14 — BEYOND TRUTH AND FALSITY

- *Ouija* board; talks with « spook »; which answers by *beep*.

  Irrational, but why? Think of an *Ipad;* what means *beep*?
  Polygraph: does not quite mean « liar », only « it matters ».
  Locative: no real contents, beyond discussion: « touché ».

- « Do you know what time it is? – Yes. » Unsatisfactory.

  Witness: would-be « proof » of answer *yes*.
  Doubt: false witness. No benchmark (pravdameter) *yes/no*.
  Witness fails to convince; need *interrogation* process.

- « Did you bring a DVD reader? — Yes, see. »

  Interrogation: Feed *reader* with DVD, use remote control.
  Remote vs. menus *language-free* dialogue.
  Witness convinces if movie actually played.
  Dialectics *witness* (reader) vs. *ordeal* (rigid DVD).

15 — HEGELIAN, A.K.A. LINEAR, NEGATION

- Relation *witness/ordeal* symmetrical: mutual agreement.

  Negation $\sim Q$ replaces answers to Q with its ordeals.
  Witnesses of $\sim Q$ are the constraints on Q.
  Hegel: contradictory foundations: Q rests on « contrary ».

- Not to be confused with usual negation $\neg A$.

  Witness: negation problematic; there is no « non-witness ».
  $\neg$ not *involutive*. Weaker, more expedient than $\sim$.
  The Godfather (1972): « a proposal that you cannot refuse ».

- According to semantic pleonasm, negation $\neg$ *refutes*.

  Hegelian (a.k.a. linear) negation *recuses*.
  Affaire Dreyfus: « la question ne sera posée ».
  Hegelian negation is a sort of *normative*, deontic prison.
  Format a.k.a. synthetic. Formation (> 0) vs. formatting (< 0).

16 — JUDGES WILL BE JUDGED

- Possible *dissensus:* reader cannot read disk.
  Bad reader? Or defective DVD? No simple way to tell.
  Sampling: restrict to generic ordeals, a finite *gabarit*.
  Reader-test and DVD-test always *accept* each other.
- *Laxism* (Volkswagen): *tested* reader may refuse *tested* DVD.
  Production $\neq$ consumption: negated by semantic *prejudice*.
  Language as well, except *ox/beef*, *calf/veal*, *sheep/mutton*.
- *Usine:* sense as question *(gabarit)*. *Proof-nets*, *cut-free*.
  A posteriori: experimental, everything checked.
  Almost analytic: but for questionable *choice* of tests.
- *Usage:* sense as use (Wittgenstein). *Indirect* answers.
  Implication $Q \Rightarrow R$: question R reduced to Q.
  Cut: answers to $Q \Rightarrow R$ and to Q perform into answer to R.
  Sampling changed by implication. Cannot stay within *usine*.

17 — THE ARCHITECTURE OF THOUGHT

- ***Richard's paradox*** **(1905); inspired Gödel's theorem.**

  Smallest integer not definable in ≤ 20 words.
  Fixed with rigourous version DEFINABLE. Not a DEFINITION.
  Refutes *qualunquism*, the analytic, « anti-format » ideology.

- **Charybdis/Scylla: *inconsistency* vs. *incompleteness*.**

  Inconsistency: format too *laxist* (original paradox).
  Incompleteness: format too *repressive* (« fixed » version).

- **Three ages of living formats: *young, senile, post-mortem*.**

  Young: protection, e.g., informatic *extensions*.
  Mathematics: stimulates, structures. Groups, morphisms.
  Senile: repressive. The russian *Tchin*. Academism.
  Apple: more and more repressive, hence the *jailbreak*.
  Post-mortem: play on the format, « second degré ».
  Has been: outed from rewarding format.

18 — ONE CAN COMPARE EVERYTHING

- **Second subliminal slogan of *transparentism*.**

  Unidimensionality: unique number, bibliomtery, QI, etc.
  Global comparison: impossible, like in *Jan-Ken-Pon*.
  Best all-times movies: reflects the jury, e.g., Brussels 1958.

- ***Complotism*, the unidimensional version of topsy-turvism:**

  Axis of Evil: Saddam, Kim, ben Laden, meet underground.
  Void of contents: only purpose seems to be *abstract* Evil.

- ***Transcription:* supposes unidimensionality.**

  Numerology: Casanova, etc. Fails even if made faithful.
  Sound to image: *Fantasia* (1940) not convincing.
  Image to sound: Xenakis (yields rumbles).
  Sound to taste: piano à cocktails (Boris Vian).
  Language to music: BACH, DSCH for Shostakovich; dubious.
  Translation: cannot render nuances in foreign language.

19 — QUALUNQUISM

- **Format is half good, half bad; unless we try to *break* it.**

  Essentialism: *conservative*, everything in its place.
  Existentialism: *protest*, counter-power.
  Complementarity: all formats *injust*; but we *need* one.

- ***L'uomo qualunque:* (ordinary man) neo-fascist party (1946).**

  Populism: le Pen, Sarkozy: *down* with politics and Justice!
  Great Leap Forward (1958): production of qualunquist steel.

- **Analytic philosophy as *tabula rasa* (Russell $\sim$1925).**

  Down with concepts: not rigourous enough! Use logic.
  Transcription problematic: how do you say « God » in logic?
  Logic not analytic. Disputable, esp. dubious *predicate* part.

- ***Declarative* (logic) programming: down with algorithmics!**

  Ad hoc: « control » ($\sim$ philosophical « logics », the *Führer*).

20 — MULTIPLICATIVE PROOF-NETS

- **Function symbols** $\mathtt{l}, \mathtt{r}, \mathtt{g}$ **(0-ary),** $\cdot$ **binary.**

  To each proposition A **associate *location*** $p_A(x)$**.**
  To each proof π **associate *vehicle*** $\pi^\bullet$**.**
  Identity axiom $\vdash A, {\sim}A$**:** $\pi^\bullet := [\![\, p_A(x), p_{{\sim}A}(x) \,]\!]$**.**

- $p_A(x) := p_{A \between B}(\mathtt{l}\cdot x)$, $p_B(x) := p_{A \between B}(\mathtt{r}\cdot x)$ ($\between = \otimes, \parr, \ldots$)

  $\parr$-rule: if π comes from ν of $\vdash \Gamma, A, B$, $\pi^\bullet := \nu^\bullet$.
  $\otimes$-rule: if π from ν, μ of $\vdash \Gamma, A, \vdash \Delta, B$, then $\pi^\bullet := \nu^\bullet + \mu^\bullet$.

- ***Ordeals:*** $q_A(x) := p_A(\mathtt{g} \cdot x)$**; the** $q_A(x)$ **pairwise *disjoint*.**

  Conclusions: green/black**, premises:** magenta/yellow**.**

- **LEGO *bricks:*** **Literals:** $[\![\, \frac{p_A(x)}{q_A(x)} \,]\!]$**; conclusion** $A \in \Gamma$**:** $[\![\, \frac{q_A(x)}{p_A(x)} \,]\!]$**.**

  $\otimes$-link: $[\![\, \frac{q_A(x), q_B(x)}{q_{A \otimes B}(x)} \,]\!]$**.**

  $\parr$-links: $[\![\, \frac{q_A(x)}{q_{A \parr B}(x)} \,]\!] + [\![\, \frac{q_B(x)}{} \,]\!]$ **or** $[\![\, \frac{q_A(x)}{} \,]\!] + [\![\, \frac{q_B(x)}{q_{A \parr B}(x)} \,]\!]$**.**

21 — CORRECTNESS

- ***Gabarit:*** **all ordeals obtained by *switching* the $\parr$-links.**

  Vehicles coloured in `blue`.
  Correctness: $\mathcal{V} + \mathcal{O}$ strongly normalises into
  Normal form: $[\![\, p_\Gamma(x) \,]\!] := [\![\, \{p_A(x); A \in \Gamma\} \,]\!]$.

- **η-expansion: identity links on literals. Criterion insensitive.**

- ***Herbrand:*** **existentials as functions of universals $\vec{y} = \vec{t}[\vec{x}]$.**

  $x := f(y)$ as independence of $y = t$ from x, i.e., $\exists y \forall x$.

- **X ($\sim X$) must be paired; not with $X, Y, \sim Y$ ($\sim X, Y, \sim Y$).**

  Essentialism: complementarity of *names*.
  Literal $X, \sim X$: occ. of *universally* quantified variable $\forall X$.
  Cancelling ordeal: special kind bound to normalise to $\emptyset$.
  Switching: select a literal in all pairs, $\sim X, \sim Y, Z$.
  Sum of all: $[\![\, \frac{q_A(\mathtt{l}\cdot x), q_A(\mathtt{r}\cdot x)}{} \,]\!]$ when literal A is selected.

22 — THE CUT RULE

- ***Lewis Carroll (1893):*** **cut identical to conclusion $A \otimes {\sim}A$.**

  Cut conclusion **with $A \multimap A$.**
  Replace **cut with $(A \multimap A) \otimes A \otimes {\sim}A$, etc.**
  Zenon: **should be the same as Achilles vs. Tortoise.**
  No paradox, just stupidity: **Achilles runs in *wrong* direction.**

- ***Cut:*** **conclusion $[A \otimes {\sim}A]$. Predicts erasure, *a priori*, usage.**

  Performance: **vehicle $\mathcal{V}$ in `blue` and `red` (for $p_A, p_{\sim A}$). Add**
  Feedback: $\mathcal{F} := [\![\frac{p_A(x), p_{\sim A}(x)}{}]\!]$.
  Elimination: **from the $\mathcal{V} + \mathcal{O}_{A \otimes {\sim}A} + \mathcal{O}$ to the $\mathcal{V} + \mathcal{F} + \mathcal{O}$.**

- ***Church-Rosser:*** **use two pairs of colours.**

  Cut-elimination: **adequation usine/usage.**
  Knitting: **compositionality, BHK.**

- **Exponentials: will involve *hidden cuts* $[\underline{A \otimes {\sim}A}]$.**

23 — IMPOSSIBLE CONNECTIVES

- **Operations not central and *poorly knitted*.**

  Exponentials: $!A, ?A$.
  Intuitionistic disjunction: $!A \oplus !B$; ***commutative*** **cuts.**
  Multiplicative neutrals: $1, \bot$.

- **These connectives only acceptable as *second-order* ones.**

  Exponentials: $!A := \forall X((A \Rightarrow X) \multimap X)$.
  Int. disj.: $!A \oplus !B := \forall X((A \Rightarrow X) \multimap ((B \Rightarrow X) \multimap X))$.
  Multiplicative neutrals: $1 := \forall X(X \Rightarrow X)$.

- **Basic problem: *weakening* impossible.**

  From Γ: **no way to derive Γ, A for any A.**
  Want **of physical connection.**
  Hidden conclusion: $\Gamma, \underline{\Delta}$.
  Ordeal: $[\![\frac{q_A(x)}{}]\!]$ **when $A \in \Delta$ hidden (variant below).**

24 — EXPONENTIALS REVISITED

- **Revert to *intuitionistic* implication. . . Not quite.**

  Bang! $A \oslash B := !A \otimes B$.
  Why not? $A \ltimes B := ?A \parr B$.

- **Vehicles: auxiliary variable for *duplication:*** $p_A(x \cdot y)$.

  Dereliction: $\vdash \Gamma, \underline{\Delta}, \underline{A}$ **from** $\vdash \Gamma, \underline{\Delta}, A$: $p_A(-) \rightsquigarrow p_A(- \cdot \mathtt{d})$.
  Weakening: **no change.**
  Contraction: $\vdash \Gamma, \underline{\Delta}, \underline{A}$ **from** $\vdash \Gamma, \underline{\Delta}, \underline{A'}, \underline{A"}$:
  $p_{A'}(- \cdot -), p_{A"}(- \cdot -) \rightsquigarrow p_A(- \cdot (\mathtt{l} \cdot -)), p_A(- \cdot (\mathtt{r} \cdot -))$.

  $\ltimes$**-rule:** $\vdash \Gamma, \underline{\Delta}, A \ltimes B$ **from** $\vdash \Gamma, \underline{\Delta}, \underline{A}, B$:
  $p_A(-) \rightsquigarrow p_{A \ltimes B}(\mathtt{l} \cdot -)$ **and** $p_B(-)) \rightsquigarrow p_{A \ltimes B}(\mathtt{r} \cdot -)$.

  $\oslash$**-rule:** $\vdash \Gamma', \underline{\Delta}, \underline{\Delta'}, A \oslash B$ **from** $\vdash \underline{\Delta}, A$ **and** $\vdash \Gamma', \underline{\Delta'}, B$:
  $p_A(-) \rightsquigarrow p_{A \ltimes B}(\mathtt{l} \cdot (- \cdot y))$ **and** $p_B(-)) \rightsquigarrow p_{A \oslash B}(\mathtt{r} \cdot -)$.

  Homogeneise **to take care of auxiliary variable and sum up.**

25 — EXPONENTIAL CRITERION

- ***Proof-nets:*** **auxiliary variable induces problems.**

  Normal form **of** $\mathcal{V} + \mathcal{O}$ **of the form** $[\![\, p_\Gamma(x) + p_\Delta(x \cdot T) \,]\!]$.
  No way **to foretell** T **(complex weakening/contraction).**
  Transcendalism: **weakening/contraction not part of answer.**

- **Criterion involves** ***non-determinism*****.**

  $A \oslash B$**:** $[\![\, \frac{q_A(x\cdot x), q_B(x)}{q_{A\oslash B}(x)} \,]\!]$ **and** $[\![\, \frac{q_A(x\cdot \mathtt{1}), q_B(x)}{q_{A\oslash B}(x)} \,]\!]$.

  Solution $q_A(x \cdot t)$ **with** t **unifying with both of** $x, \mathtt{1}$**:** $t = y$.
  $A \ltimes B$ **sort of** $\parr$ **without left switching.**

  $\ltimes_R$**:** $[\![\, \frac{q_B(x)}{q_{A\ltimes B}(x)} \,]\!] + [\![\, \frac{q_A(x\cdot y)}{} \,]\!] + [\![\, \frac{q_A(x'\cdot y')}{} \,]\!]$ $(x \neq x')$.

  Solution $q_A(t_i \cdot u_i)$ **would produce duplicate if** $t_i \neq x$.

  $\ltimes_L$**:** $[\![\, \frac{q_A(x\cdot y)}{q_{A\ltimes B}(x\cdot y)} \,]\!]$ **(cancelling).**

  Impossibility **to reach premise** A **« from below ».**

III — WHAT CONVEYS CERTAINTY?

Keywords: derealism, epidictic, épure.

26 — HILBERT OUT OF FOCUS

- ***Axiomatics:*** **in modern greek, *officer*, not quite logical!**

  XIX$^{\text{th}}$ century: axioms + *Modus Ponens* (usage), no usine.
  Mistakes: located in « false » axioms; no *pravdameter*.

- **Sort of usine: *limited* questions $2+2=4$ or $2+2=5$.**

  Consistency: axioms should not yield *incorrect* $2+2=5$.
  Kant « fixed » by Hilbert: *consistency* of presuppositions.
  Scientistic *self*-justification of science.

- ***So far so good:*** **consistency not analytic (not performative).**

  Incompleteness: neither *checkable* nor *provable*.
  Inconsistency analytic : performative, *expansive*.

- ***Confidence*** **not ensured by consistency.**

  Inconsistency consistent? *Indirect* proof procrastinates.
  Never seen never taken: a sort of logical *dismissal*.

27 — ON THE TRAIL OF THE DOUBT

- **Axiomatic *smoothing:* tree-like form *inexpressive*.**

  Realism: *Modus Ponens* preserves *ethereal* truth.
  Usine: axioms can usually be *checked*.
  Modus Ponens problematic, involves change of *gabarit*.

- ***The fly (1986):* neither man nor fly, nor both! *Mix* man+fly.**

  Imbrication: $Q \multimap R$ imbricates questions $\sim Q$ and R.
  Sequent $\vdash Q, R$ imbricated questions.
  Extension wire $\vdash \sim Q, Q$ production/consumption of 127V.

- **« The medium is the message »: sense is form, shape.**

  Proof-nets trees imbricated through *paired leaves*.
  Travel not tree-like; conveys actual *semantic-free* meaning.

- ***Desimbrication*. Recover man from mix *man+fly*.**

  Lewis Carroll imbricates! Need *Cut* with « anti-fly ».

28 — RIGHTS AND DUTIES

- Cut involves a *performance*; may diverge (procrastinate).

  Laxist *gabarits:* Volkswagen.

- Mismatch usine/usage (*Prawitz:* introduction/elimination).

  Usine: the *right* to use a name.
  Usage: the corresponding *duties*.

- Mismatch comes from *incomplete* gabarits.

  Perfect case: (multiplicatives, etc.) *possible* completion.
  Imperfect case: (exponentials + second order) *impossible*.

- *Popper:* use incomplete gabarits; « so far so good ».

  Fitted for *medicine*, since non deductive.
  Empirism: restricted to *reproduction*.
  Unfitted for prevision, deduction. « Butterfy effect ».
  Gabarits deeply altered by *indirect*, deductive answering.

29 — DEREALISM

- **Avoid pitfall of infinite gabarit by *symbolic* testing.**

  Recurrence involves a ***reduction*** of test $n+1$ to test n.
  Second order quantification in Dedekind definition of $\mathbb{N}$.
  Proof-net: existential $\exists X\, A$ involves ***witness*** T in $A[T]$.
  T is indeed a ***synthetic*** component of the answer.

- ***Derealism*** **not Object/Subject: answer partly *subjective*.**

  Épure: combination vehicle + gabarit. Object + ***look*** at it.

- **Gabarits come by *pairs* $T, \sim T$; are they *balanced*?**

  Gabarit/vehicle: similar to police/yakuza.
  Derealism: some police in the role of yakuza.
  Conflict of interest: gabarit-witness has rather be laxist.

- ***Apodictic*** **(literally, proven): *legitimate* certainty impossible.**

  Epidictic: ***reasonable*** certainty; belief in ***balanced*** gabarits.

30 — ONE CAN PREDICT EVERYTHING

- **Third subliminal *transparentist* slogan: negates doubts.**

  Date of death known *in advance*: paradoxical.
  Subjective break: *wrong news* in to-morrow's paper.
  Retrodiction: Nostradamus w.r.t. death of Princess Diana.

- **Prediction in *conditional* tense.**

  Counterfactuals if...Parallel models à la Kripke.
  Conditional premise: stands as *joke* or result of *sake*.
  If I married your mother in 1946, I would now be your *father*.
  Sanma no aji (1962): if Japan *had won* the war, then...

- ***Inverse reasoning* in mathematics yields *conjectures*.**

  Abduction: all conjectures true.
  If $A \Rightarrow B$ then $B \Rightarrow A$.
  Restriction: to be used when it works, i.e., *never*.
  Sherlock Holmes: Conan Doyle selects *relevant* clues.

31 — HEGEL AND CONSISTENCY

- ***Paraconsistent:*** **un-inconsistent, i.e., *undead*.**

  Vampires: good for nothing: don't *reflect* in mirrors.
  Adequacy u/u fixed by killing usage: no *consequence*.
  Typical « Theorem »: all integers even and equal to 29.

- **Originates in Brazil, with plausible influence of *terrorism*.**

  Shindô Renmei: paraconsistant *victory* of Japan ($\sim$ 1946).
  Rubber cheque: acceptable only at the point of a gun.

- **Hegel *mistreated* in $\mathrm{xx}^{\mathrm{th}}$ century: nazi & paraconsistent.**

  Contradictory foundations require answer to *any* question.
  Derealist explanation: épures part of general *animæ*.
  Anima: mingles Object and Subject, cannot be split $\mathcal{V} + \mathcal{G}$.

- **0 admits *animist* proofs: nightmare of *empty* types fixed.**

  However $A, \neg A$ cannot both have *non-animist* proofs.

32 — CUT-ELIMINATION

- ***Vehicle*** $\mathcal{V}$ **with conclusions** $\vdash \Gamma, [A \otimes {\sim}A]$ **and**
  Feedback: $\mathcal{F}_A := [\![\frac{p_A(x), p_{\sim A}(x)}{}]\!]$; **fits** $p_A(-)$ **and** $p_{\sim A}(-)$.
  Performance: $\mathcal{V} + \mathcal{F}_A$ **possibly yields normal form** $\mathcal{W}$.
  Correctness of $\mathcal{W}$ **w.r.t.** ***ordeal*** $\mathcal{O}$ **for** $\vdash \Gamma$.
- **Case** $A = X$: $\mathcal{V} = [\![\frac{}{p_{\sim X'}(x), p_X(x)}]\!] + [\![\frac{}{p_{\sim X}(x), p_{X''}(x)}]\!] + \ldots$.
  Then: $\mathcal{W} = [\![\frac{}{p_{\sim X'}(x), p_{X''}(x)}]\!] + \ldots$ **passes test** $\mathcal{O}$.
- **Case** $A = B \otimes C$; **replace** $\mathcal{F}_A$ **with** $\mathcal{F}_B + \mathcal{F}_C$.
  Change of syntheticity: two cuts $\vdash \Gamma, [B \otimes {\sim}B], [C \otimes {\sim}C]$.
  $\mathcal{V} + \mathcal{F}_A$ **same normal form as** $\mathcal{V} + \mathcal{F}_B + \mathcal{F}_C$.
- **Replacing** $[\![\frac{q_D(x)}{p_D(x)}]\!]$ **with** $[\![\frac{q_D(x)}{}]\!]$ **in** $\mathcal{O}$ **yields** ***closing*** $\mathcal{O}'$.
  Main result: $\mathcal{V} + \mathcal{O}'$ **normalises into:**
  $[\![\frac{}{p_B(x)}]\!] + [\![\frac{}{p_C(x)}]\!] + [\![\frac{}{p_{\sim B}(x), p_{\sim C}(x)}]\!]$.

33 — EXPONENTIAL CUTS

- **Cut on $A = B \oslash C$.**

  $\mathcal{V} + \mathcal{F}_A$ same normal form as $\mathcal{V} + \mathcal{F}_B + \mathcal{F}_C$.
  Same as: $\mathcal{V} + \mathcal{F}_B \otimes t_1 + \cdots + \mathcal{F}_B \otimes t_n + \mathcal{F}_C$.
  Choice between: $p_B(x) \otimes y := p_B(x) \cdot y/ := p_B(x \cdot y)$?
  Knitting: second solution enables change of syntheticity.

- **Multiplicative cut-elimination works *mutatis mutandis*.**

  Cut on A replaced with several cuts (C and copies of B).
  Copies not well defined: may change with switching.

- **Problem when *resuming* cut-elimination.**

  Unrelated switchings of the cuts $[B \otimes {\sim}B] \otimes t_i$.
  Non-deterministic sum of all switchings of $B \otimes {\sim}B$.
  Independence: when auxiliary parameters y, y' distinct.

34 — SYSTEM F

- ***Second order*** **quantifications: over** ***propositions*****.**

  Links:

$$\frac{A}{\forall X\, A} \qquad\qquad \frac{A[T/X]}{\exists X\, A}$$

- **Can be handled by** ***usine*** **(proof-nets).**

  $\forall X$**:** $X := \cdot/ \otimes /\parr$**, hence** $\sim X := \cdot/ \parr /\otimes$**.**

  Existential $\exists X$**:** T **provides its** ***own*** **switchings.**

- **However,** T **is part of the** ***derealist*** **answer.**

  Épure: combination vehicle + ***mould*****, e.g.,** $T + \sim T$**.**

  Balance: how do we know that $T + \sim T$ **actually** ***match*** **?**

  Object/Subject no longer valid: answer partly ***subjective*****.**

  Answer combines ***analytic*** **and** ***synthetic*** **features.**

  Epidictic: uncheckable affirmation. $\neq$ ***apodictic*****.**

35 — ANIMÆ

- **Derealism: two pairs, `blue`/`yellow` and `red`/`cyan`.**

  Animæ: uses colours `blue`, `red`.
  Épure: splits as $\mathcal{V} + \mathcal{M}$.
  Animist otherwise: Object and Subject *intertwined*.
  Ordeal: uses colours `yellow`, `cyan`, `black`.

- ***Additive neutrals:* no balance problem in $\exists X\, X$.**

  $\top$: ordeal $[\![\frac{R(x),S(x)}{}]\!] + [\![\frac{T(x)}{\top(x)}]\!]$ and cancelling $[\![\frac{R(x),S(x)}{\top(x)}]\!]$.

  0: three ordeals, $[\![\frac{r(x)}{}]\!] + [\![\frac{s(x),t(x)}{O(x)}]\!]$ and

  $[\![\frac{s(x)}{}]\!] + [\![\frac{r(x),t(x)}{O(x)}]\!]$ and $[\![\frac{t(x)}{O(x)}]\!]$.

- **The absurdity has an *animist* proof:**

  $[\![\frac{}{t(x)}]\!] + [\![\frac{}{r(x),s(x)}]\!]$.

  But no épure: hence consistency.

IV — LA FOUTUE RÉALITÉ

36 — THE EXPULSION OF SUBJECT

- ***Subjectivistic*** **paranoïa:** ***exagerates*** **syntheticity.**
  Number 13: rename into row 14. Complotism.
  Causality subjectivistic: butterfly cannot ***cause*** **storm.**
- ***Objectivistic*** **schizophrenia:** ***negates*** **syntheticity.**
  Left-handed cups.
  Ptolemaic astronomy: parallax objectivised into ***epicycles*****.**
- **Causality, consequence,** ***subsequence*****.**
  Cause before ***effect*****; hence « subsequence ».**
  Saint Anthony patron saint of subsequence.
- **Causality** $\neq$ **consequence.**
  I am living $\Rightarrow$ **I was born. Not a causality!**
- ***Objectivisation*** **of consequence.**
  Possible worlds: Leibniz equality.

37 — MISERY OF SEMANTICS

- **Semantic dogma of *subsequence*.**
  Consequence reduced to factual justification.
  Aristotle: *paralogism*. However, accepted factual *refutation*.
- **Non euclidian geometries: Euclid's postulate.**
  Sphere: no parallels; *atomic plant*, too many.
  Much better than cognitive investigation; but *accidental*.
- **Deficit of reality with $\mathbb{N}$.**
  No realist explanation of *absence* of consequence.
  Only one universe of integers (*Kronecker*).
  Non standard integers, sorts of *epicycles* of realism.
  Selfy of incompleteness. Should be other way around.
  Not analytic: out of reach, not computable.
- ***Derealism:* the look at an *object* part of the object.**
  Épure: combination vehicle + *mould*.

38 — REALITY AS KNITTING

- ***Chicken and egg*** **dilemma: search for** ***objectivity*****.**
  Ohm's law: $U = R \times I$**.**
  Enables measures of tension, resistance.
  Verifies law through artifact made according to law.
- **Abstractions as** ***ideal*** **limits: measure of** ***tension*****.**
  $U = (r + R) \times I = r \times I + R \times I$**; if ratio** r/R **« small ».**
  Actual U **obtained as limit** $R \to \infty$**.**
- **No hen/egg, only** ***knitting*** **constat/performance/usine/usage.**
  Reality: the forgetting of the knitting.
- ***Realism:*** **the forgetting of the forgetting.**
  Leads to identifications implicit/explicit, analytic/synthetic.
  Alternative refusal of ***performance:*** **pravdameter or**
  Usine: (non monotonicity) or
  Usage: (paraconsistency).

39 — ABOUT DOUBT

- Cannot doubt of ***everything***, e.g., that I wrote `wrote`.
- ***Reasonable*** doubts: as to medicine and ***empirical*** activities.
  Empirism expresses doubts; but ***generates*** them!
  Repetitivity: ***same*** (close) causes yield ***same*** (close) effects.
  Approximate testing (Popper) not predictive; the ***butterfly***.
  Lourdes confirms unreliability of medicine.
- ***Reasonable*** certainty: suspended doubts.
  Legitimate doubts due to ***derealistic*** features.
  Deductive method replaces ***inductive*** empirism.
  Understanding: knitted knowledge. ***Mathematics***.
  4-colours proof not knitted enough for mathematical taste.
  Quine's NF refused because not knitted to mathematics.
- No knitting ***criterion*** (cut-elimination, Church-Rosser).
  Science: only produces the best knitting ***so far***.

40 — THE CONTROVERSIAL PREDICATES

- System $\mathbb{F}$: propositions are (roughly) enough.
  Forgetful functor: keeps computational (analytic) contents.
  Realisability: awkward reduction predicate $\rightsquigarrow$ proposition.
  Drop in quality when passing from boolean to ***cylindric***.
- ***Predicate calculus:*** XIX$^{\text{th}}$ century legacy.
  Axiomatics: cannot avoid « **Barbari** » $\forall x\, A \vdash \exists x\, A$.
  Semantics: models non-empty; but justification empty.
- Dubious principle: besides ***proper*** variables, used for $\vdash \forall$
  Junk variables: dedicated to the sole ***Barbari.***
- Intrusion of reality through ***external*** domain.
  Variables, functions: proceed from the Sky.
- In constrast to ***propositional*** quantification:
  Variables: refer to propositions, well-defined by l'usine.
  Functions: refer to connectives.

41 — EQUALITY

- ***Logical*** **primitive mistreated by metaphysical** ***axiomatics:***

  E.g., a predicate: « function » individuals $\rightsquigarrow$ propositions.

- **And/or through** ***semantic*** **pleonasm:**

  BHK: empty, reduces proof of $t = u$ to semantics.
  Semantics: $t = u$ true when ***same*** **denotation: $|t| = |u|$.**

- $\forall X\,(Xt \Rightarrow Xu)$ **(Leibniz) interesting,** ***since*** **totally wrong.**

  2nd order: not expected at elementary level.
  Circular: are those two « c » equal? Prejudiced:
  Relevant properties: those compatible with. . . equality.

- **A logical** ***epicycle*****, i.e., a realistic contraption.**

  Individuals + predicates: ***all*** **of those which are** ***relevant.***

- **Break epicycle by replacing** ***individual*** t **with** ***proposition*** t**.**

  Meaning: « I am t ». Equality as logical equivalence $t \equiv u$.

42 — INDIVIDUALS AS MULTIPLICATIVES

- ***Individuals = proposition*** **forbidden by prejudice:**

  Classical: $t \equiv u \vee u \equiv v \vee v \equiv t$**. Only two individuals.**
  Intuitionistic: $\neg\neg(t \equiv u \vee u \equiv v \vee v \equiv t)$**. Not more than** 2**.**
  Linear: with $(t \multimap u)\ \&\ (u \multimap t)$ **as equality. No obstacle.**

- n**-ary multiplicative: sets of partition of** $\{1, \ldots, n\}$**.**

  Duality: $\mathcal{C} \perp \mathcal{D}$ **iff their incidence graph is a tree** $(n \neq 0)$**.**
  Multiplicative: non-trivial set of partitions equal to bidual.
  Example: $\otimes := \{\{1,2\}\}$ **vs.** $\parr := \{\{1\},\{2\}\}$**.**
  Series/parallel: $\P := \{\{1,2\},\{3,4\}\} + \{\{2,3\},\{4,1\}\}$**.**
  Not sequential: $\P$ **admits proof-nets, no sequent calculus.**

- ***Linear*** **implication between multiplicatives:**

  Same n **: typically,** $* \otimes (* \parr *) \multimap (* \otimes *) \parr *$ **with** $n = 3$**.**
  ♯ partitions: decreases; equal in case of equivalence.
  Equality: equivalence yields two *isomorphisms*, not related.

43 — FUNCTIONS AND PREDICATES

- **Functional *terms* come from same multiplicative matrix:**

  Positive multiplicatives with possible repetitions.
  Example: $x \parr (x \otimes y)$. No constant, no *Barbari*, no regrets.
  Pairing: ensured by $(x \parr y) \otimes (x \parr x \parr y)$.

- ***Predicate* variables $P, Q, \ldots$ as variable *connectives.***

  Pt handled by unknown binary connective K.
  Usage: all possible uses $Kt\tilde{t}$ of individual t and negation $\tilde{t}$.
  Usine: enough to test with $K = \otimes$ and $K = \parr$.
  Equality principle: $t = u \Rightarrow (Pt \multimap Pu)$ OK'ed by l'usine.
  Refused: $t = u \Rightarrow (Pt \multimap Qu)$ and $t = u \multimap (Pt \multimap Pu)$.

- ***Equality* handled by: $(\tilde{t} \parr u) \,\&\, (t \parr \tilde{u})$.**

- **First-order quantification: restriction of « full » case.**

  Existential witnesses: taken among multiplicative terms.

44 — DISCUSSION

- **Logic is second order, including so-called first-order:**

  Propositions: variables, implicit $\forall X$ performed after.
  Usage: externalised by counter-models ($\exists X$ forbidden).
  No testing: dubious advantage of externalisation.

- **Individuals: *tame* second order. No derealism.**

  Witnesses: multiplicatives, limited loss of subformula pty.
  Balance: rights/duties, usine/usage not really problematic.

- ***Arithmetic:* all axioms removed but:**

  Third/fourth Peano axioms: $Sx \neq 0$ and $Sx = Sy \Rightarrow x = y$.

- **The origin of logical doubt (incompleteness, etc.):**

  Épure vs. gabarit: performance $\mathcal{V} + \mathcal{M} + \mathcal{G}$.
  Variance: usine works better with lax $\mathcal{M}$. Usage may fail.
  Example: induction on « ill-formed » $\mathcal{M}$.

45 — ANTI-CLASSICAL PROGRAM

- **Idea: sever all *bridges* with semantics.**

  Refute classical principles, e.g., weakening/contraction.

  $\neg\forall X\forall Y((X \otimes Y) \Rightarrow X)$ **and** $\neg\forall X(X \Rightarrow (X \otimes X))$**.**

- **Expected outcome: increase in *logical* expressiveness.**

  Natural numbers: complete *logicisation* of arithmetic.

  $\overline{m} \neq \overline{n}$ **(for** $m \neq n$**) not provable in linear logic.**

- **Unfortunate « classical » forgetful functor.**

  Clue: use *non sequential* connectives, e.g., ¶.

  Semantics: inexistant. Indeed, intersection types.

$$¶(A, B, C, D) = ((A \otimes B) \parr (C \otimes D)) \cap ((B \otimes C) \parr (D \otimes A)).$$

$$\sim¶(\sim A, \sim B, \sim C, \sim D) = ((A \otimes C) \parr (B \parr D)) \cap ((A \parr C) \parr (B \otimes D)).$$

- **Conjecture: find a classical inconsistent multiplicative.**