CSL PARIS 2001

FROM THE RULES OF LOGIC TO THE LOGIC OF RULES

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Locus Solum

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1-The role of negation

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Le jeu de Bonneteau

♡ ♠ ♢

Question : where is the meaning ?

Le jeu de Bonneteau

♡ Syntaxe

♠ Sémantique

♢

Le jeu de Bonneteau

♡	♠	♢
Syntaxe	**Sémantique**	**Méta**
τάξις	*σῆμα*	*μέτα*

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- **Not to know vs. to know not ; everything in between.**

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- **Logical time as alternation positive/negative.**

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- **$\mathfrak{D} \perp \mathfrak{E}$ iff $\mathfrak{D} = \maltese$ or $\mathfrak{D}$ starts with $(+1, \xi, \mathrm{I})$, $I \in \mathcal{N}$.**

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  $\forall \mathbf{X}(\mathbf{A}\mathbf{X} \vee \mathbf{B}\mathbf{X}) \Rightarrow \forall \mathbf{X}\mathbf{A}\mathbf{X} \vee \forall \mathbf{X}\mathbf{B}\mathbf{X}$.
- Extend to exponentials.
- Low complexity integers ?

10-GREAT EXPECTATIONS

- Define **locative** type theories.
- New types, e.g., **projections**.
- **Prenex forms**. $\exists \mathbf{X} \forall \mathbf{Y}\ \mathbf{A}\mathbf{X} \Rightarrow \mathbf{A}\mathbf{Y}$.
  $\forall \mathbf{X}(\mathbf{A}\mathbf{X} \vee \mathbf{B}\mathbf{X}) \Rightarrow \forall \mathbf{X}\mathbf{A}\mathbf{X} \vee \forall \mathbf{X}\mathbf{B}\mathbf{X}$.
- Extend to exponentials.
- Low complexity integers ?
- Can we get rid of the **ambiant** classical logic ?