LOCUS
SOLUM

Jean-Yves Girard

1   DUALISM IN LOGIC

DUALISM

▶ **Schizophrenia ideas/matter. Descartes.**

▶ **In logic :**

- **Properties vs. Objects.**

- **Syntax vs. semantics.**

▶ **Tarski.**

TARSKIAN SEMANTICS

▶ $A \wedge B$ is true when A is true and B is true.

▶ $A \vee B$ is true when A is true or B is true.

▶ $\neg A$ is true when A is not true.

▶ $\forall x A[x]$ is true when $A[c]$ is true for all c.

LA PALICE

- **Un quart d'heure avant sa mort, il était encore en vie.**

- **vs. Truth is the quality of what is true.**

- **Unfair, Tarski invented meta : compare $\wedge$ and and.**

- **Un quart d'heure avant sa mort, il remarqua qu'il était encore en vie.**

THE HOLY TRINITY

► **Syntax** $\tau\acute{\alpha}\xi\iota\varsigma$

► **Semantics** $\sigma\tilde{\eta}\mu\alpha$

► **Meta** $\mu\acute{\epsilon}\tau\alpha$

ARISTOTLE

- No distinction **syntax/semantics.**

- **Barbara :** every A is B, every B is C, hence every A is C.

- **Scholastic interpretation : reduce Barbara to Disamis...**

- **Tarski : if** $A \subset B$ **and** $B \subset C$, **then** $A \subset C$.

- **The real scholastics : Tarskism.**

2   TOWARDS MONISM

GÖDEL'S INCOMPLETENESS

▶ **Second theorem : you cannot fix your spectacles while wearing them.**

▶ **First theorem : mismatch between provability and truth, between the world and its representation.**

▶ **Internal version : questioning about the reality of reality.**

BROUWER

▶ **Mathematics eventually refers to itself.**

▶ **Identity between the world and its representation.**

▶ **Commutation between provability and connectives, e.g.,**

if $A \vee B$ is provable, then one of A or B is provable.

▶ **Intuitionistic logic.**

INCOMPLETENESS REVISITED

▶ **Provability does not commute with negation.**

BLACK MASS

- **Paralogics make negation commute with provability.**

- **To know not = Not to know : joke of the prisoners.**

- **Plain logical revisionism.**

ORTHOGONALITY

▶ **Make negation commute with provability.**

▶ **A proof of $A^{\perp}$ is an object orthogonal to all proofs of A.**

3 FORMULAS VS. PROOFS

CLASSICAL LOGIC

► **Logic of reality**

► **Rules established in XIXth century.**

PROOF-THEORY

- ▶ **Study of the rules of logic.**

- ▶ **To make consistency proofs.**

- ▶ **Insurance against the explosion of Earth.**

- ▶ **Gentzen.**

CONSTRUCTIVITY

▶ **Merge between Proof theory and Intuitionism.**

▶ 1970**, Curry-Howard isomorphism.**

▶ **Proofs-as-functions.**

▶ **Proofs as morphisms.**

PROCEDURAL LOGIC

- **Intuitionistic logic, Linear logic.**

- **Logic is about itself, refers to its rules.**

- **From the rules of logic to the logic of rules.**

FORMULAS VS. PROOFS

▶ **Formula as set of its proofs.**

▶ **Vacuum cleaner : there should be enough proofs.**

▶ **Proofs with mistakes, paralogisms.**

▶ **Typical exemple, the Daimon.**

$$\frac{\qquad}{\vdash \Lambda} \maltese \qquad\qquad (1)$$

4   DESIGNS

LOCUS SOLUM

▶ **What remains of a proof when we have forgotten**

- **The formula proven.**

- **Irrelevant temporalities ; focalisation.**

▶ **Only the location matters.**

▶ **Designs can also be seen as sort of models.**

POLARITIES

▶ **Linear logic distinguishes**

- **Positive operations : multiplicative conjunction $\otimes$, active.**
- **Negative operations : additive conjunction $\&$, passive.**

▶ **Positive rules vs. negative rules.**

LOCI

- **Locus : Location ξ of a formula, e.g. address in computer.**

- **Bias : Relative location i of immediate subformula $\xi * i$.**

- **Ramification : Finite set I of biases. Corresponds to a positive rule (ξ, I).**

- **Directory : Set $\mathcal{N}$ of ramifications . Corresponds to a negative rule $(\xi, \mathcal{N})$.**

RULES

►

$$\frac{\ldots \xi * i \Vdash \Lambda_i \ldots}{\Vdash \Lambda, \xi} \; (\Vdash \xi, I) \tag{2}$$

►

$$\frac{\ldots \vdash \Lambda_I, \xi * I \ldots}{\xi \vdash \Lambda} \; (\xi \Vdash, \mathcal{N}) \tag{3}$$